Analysis of a Centered-Inclined Waveguide Slot Coupler

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Abstract —A rigorous analysis of a centered-inclined broad wall slot coupler between two crossed rectangular waveguides is presented. Pertinent integral equations are developed, taking into account finite wall thickness. The integral equations are then solved for the slot aperture E field using the method of moments. Coupling slot characteristics are deduced, including resonant length and dominant mode scattering in both waveguides. Numerical results for resonant length and scattering parameters are presented over a range of tilt angles, frequencies, and waveguide dimensions. These results have significant application in the design of waveguide-fed slot arrays.

I. Introduction

WAVEGUIDE-FED planar slot arrays often employ resonant slots to couple power from the main waveguide to crossed branch waveguides. A centered-inclined slot located in the common broad wall is a widely used coupling element for this purpose. This coupler is called a series–series slot since it can be represented by a series impedance in the branch line waveguide as well as in the main waveguide. Power coupled into the branch waveguide is controlled by the magnitude of the tilt angle, θ, with respect to the longitudinal axis of the main line. The amplitudes of the waves propagating away from the slot in the two directions in the branch line are the same, but the phases differ by 180°. For $\theta = 0°$, there is no coupling and for $\theta = 45°$ half the power is coupled into the branch waveguide. For $\theta < 0°$, the branch line fields exhibit a 180° phase difference with respect to a slot for which $\theta > 0°$.

Another commonly employed coupling slot is longitudinal and offset from the broad wall center line in the main waveguide and centered transverse in the branch waveguide. This coupler behaves very nearly as a shunt element in the main waveguide and as a series element in the branch waveguide. For the shunt–series coupler the amount of coupling is dependent on the offset in the main waveguide.

Slot couplers between waveguides were originally studied by Watson [1], who derived approximate expressions for several equivalent circuit representations. Waveguide couplers have since been discussed in the literature by many authors [2]–[6]. A rigorous analysis of slot couplers using an integral equation formulation was first presented by Vu Khac and Carson [7]. They solved the pertinent integral equations by the method of moments and obtained numerical results for longitudinally offset and transverse slot couplers. Park *et al.* [8] investigated the shunt–series coupling slot using a similar technique. Senior studied the problem of higher order mode coupling between a shunt–series coupling slot and a pair of straddling radiating slots in the branch waveguide [9]. A compound radiating slot, offset from the center line and tilted with respect to the longitudinal axis of the waveguide, has been analyzed by Rengarajan [10], who developed integral equations for the slot E field, taking into account finite wall thickness, with a solution obtained using the method of moments. A similar technique is employed in this paper for the centered-inclined slot coupler.

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II. Analysis

Fig. 1 shows two rectangular waveguides, a branch line guide designated by ports 3 and 4 and a main line guide (ports 1 and 2) situated underneath and perpendicular to the branch line. A centered coupling slot is cut in the common broad wall between the two waveguides. The slot is tilted at an angle θ with respect to the longitudinal axis of the main waveguide; it has a length $2l$ and a width w. Both waveguides are assumed to be composed of perfectly conducting walls and to be filled with homogeneous, isotropic lossless dielectrics whose constitutive parameters are μ_0, ϵ_m and μ_0, ϵ_b in the main and branch lines, respectively. The branch guide cross-sectional dimensions may be different from those of the main guide. The common broad wall thickness t between the lower and upper slot apertures will be referred to as the wall thickness in further discussions.

Upon invocation of Schelkunoff's equivalence principle the domain of this coupling slot problem is divided into three regions: the main waveguide interior, the branch waveguide interior, and a rectangular cavity of dimensions $2l$, w, and t, as shown in Fig. 2. For the purpose of solving the problem in the main waveguide interior (region 1), electric and magnetic equivalent currents in the lower slot aperture region are considered. A perfect conducting short is then placed in the slot aperture at $y = b$ to suppress the equivalent electric currents. A magnetic current sheet $\bar{K}_m^{(1)}$

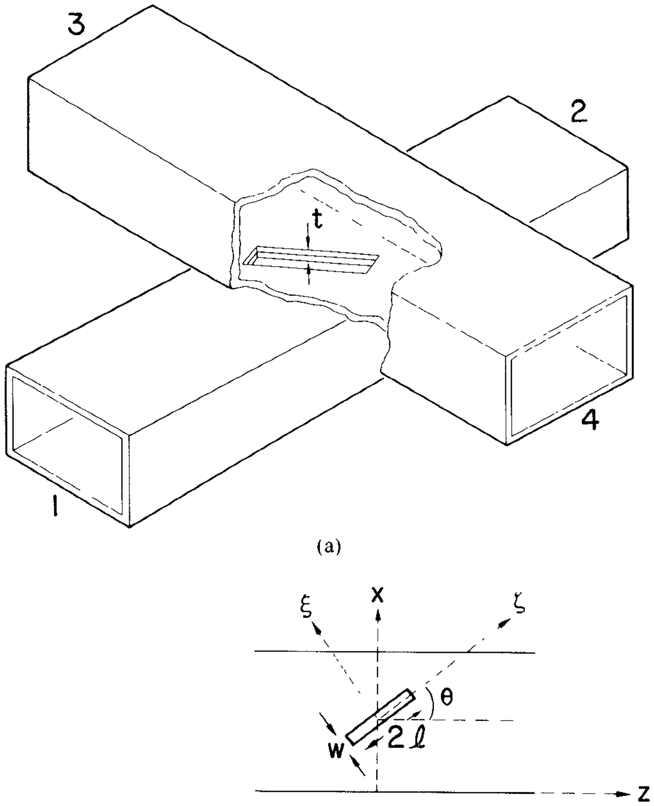

Fig. 1. (a) Geometry of a centered-inclined coupling slot. (b) Coordinates in the upper broad wall of the main waveguide.

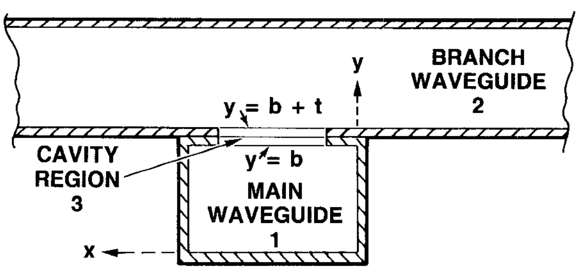

Fig. 2. Waveguides and the cavity region.

placed at $y = b^-$ produces the scattered fields in the main guide. In addition, a TE_{10} mode wave incident at port 1 excites this waveguide. A similar procedure is employed to solve the problem in the branch waveguide interior (region 2), where the slot aperture is shorted by a perfect conductor and a magnetic current sheet $\overline{K}_m^{(2)}$ is placed at $y = b + t^+$. In the cavity region also both slot apertures are shorted. Magnetic equivalent current sheets $-\overline{K}_m^{(1)}$ and $-\overline{K}_m^{(2)}$ at $y = b^+$ and at $y = b + t^-$, respectively, produce the fields in region 3. The magnetic currents are surrogates for the aperture electric fields. The longitudinal component of the aperture electric field is neglected. This assumption is justified for narrow slots. Thus the magnetic currents are $\overline{K}_m^{(1)} = \hat{\zeta} E_{a\xi}^{(1)}$ and $\overline{K}_m^{(2)} = \hat{\zeta} E_{a\xi}^{(2)}$, where $E_{a\xi}^{(1)}$ and $E_{a\xi}^{(2)}$ are the transverse electric field components of the lower and upper apertures, respectively.

A pair of coupled integral equations is obtained by enforcing the continuity of longitudinal magnetic fields across each slot aperture. These equations are

$$H_\zeta(P_1^-) = H_\zeta(P_1^+) \tag{1a}$$

$$H_\zeta(P_2^-) = H_\zeta(P_2^+). \tag{1b}$$

Here P_1^- and P_1^+ have the same ξ, ζ coordinates in the lower slot aperture. The superscript $-$ refers to "just inside" the waveguide interior and the $+$ sign refers to "just inside" the cavity. Similarly, P_2^- and P_2^+ are points "just inside" the cavity and "just inside" the branch waveguide interior, respectively. Equations (1) can be rewritten as

$$H_{\zeta 1}^c - H_{\zeta 1}^{\text{scat}} = H_\zeta^{\text{inc}} \tag{2a}$$

$$-\overline{H}_{\zeta 2}^c + H_{\zeta 2}^{\text{scat}} = 0. \tag{2b}$$

H_ζ^{inc} is the longitudinal magnetic field in the lower slot aperture due to a TE_{10} mode source incident at port 1 of the main waveguide [10], and $H_{\zeta 1}^{\text{scat}}$ is the longitudinal magnetic field in the aperture region of the main waveguide interior, scattered by the slot. This scattered field is in terms of an integral involving the aperture magnetic current $K_{m\zeta}^{(1)}$ and Stevenson's Green's functions [10]. $H_{\zeta 1}^c$ is the cavity magnetic field in the lower aperture region due to magnetic current sheets $-\overline{K}_{m\zeta}^{(1)}$ and $-\overline{K}_{m\zeta}^{(2)}$. $H_{\zeta 2}^c$ is the upper aperture magnetic field in the cavity due to the above-mentioned magnetic currents. The cavity fields are in terms of integrals involving the cavity Green's functions [10]. $H_{\zeta 2}^{\text{scat}}$ is the field in the aperture region of the branch waveguide interior scattered by the slot. This scattered field is in terms of an integral involving $K_{m\zeta}^{(2)}$ and Stevenson's Green's functions for the branch waveguide with a tilt of $90° - \theta$ [10]. Thus, equations (2) represent a pair of coupled integral equations in two unknowns, $K_{m\zeta}^{(1)}$ and $K_{m\zeta}^{(2)}$.

The integral equations have been solved by the method of moments. A global Galerkin technique embodying pulse functions to represent transverse distribution and sinusoidal longitudinal distribution has been employed in this work. Testing functions involving a Dirac delta function transversely and sinusoidal variations longitudinally have been chosen to produce a matrix set. This technique has the advantage that a few expansion modes are usually adequate to ensure a good solution near resonance. The matrix set for this coupling slot problem differs from the previously used compound slot matrix [10, eq. (20)] in that $Y_{p,q}^{\text{ext}}$ now becomes $Y_{p,q}^{\text{int}}$ in terms of the branch waveguide parameters with a tilt of $90° - \theta$. Additionally, compared to the previous work (cf. [10, eq. (15)]), some simplifications result from the centering of the slot. Still the Y_{pq}^{int} expression used here consists of a double summation of a large number of algebraic, trigonometric, and exponential functions [11]. These expressions are not presented in this paper due to space constraints. Once the matrix equations are solved, the electric fields in the upper and lower apertures are known. The dominant mode scattering in the main waveguide and branch waveguide is then determined using the Stevenson's Green's function for the TE_{10} mode

[11]. For the computations to be discussed, an air-filled main line and branch line having identical cross sections have been chosen. However, the analysis is general and therefore valid for dissimilar waveguides.

III. Numerical Results and Discussion

The scattering parameters of a resonant slot coupler referenced to the center of the slot can be represented in the matrix form

$$S(\theta)=\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14}\\ S_{21} & S_{22} & S_{23} & S_{24}\\ S_{31} & S_{32} & S_{33} & S_{34}\\ S_{41} & S_{42} & S_{43} & S_{44}\end{bmatrix}. \quad (3)$$

Because of symmetry and unitary properties, the above-mentioned matrix can be expressed in terms of only one unknown, i.e.,

$$S(\theta)=S_\theta(r,p,q)=\begin{bmatrix} r & p & q & -q\\ p & r & -q & q\\ q & -q & p & r\\ -q & q & r & p\end{bmatrix} \quad (4)$$

where $r = S_{11}(\theta)$, $p = 1 - S_{11}(\theta)$, and $q = |[S_{11}(1-S_{11})]^{1/2}|$. Also, from symmetry considerations it is found that

$$S(-\theta)=S_\theta(r,p,-q) \quad (5)$$

and

$$S(\pi/2-\theta)=S_\theta(p,r,q). \quad (6)$$

Thus a θ range from 0° to 45° is adequate to characterize the coupling slot. For all cases studied in this work, representation of the slot as a *series resonant element* in both waveguides has been found to be excellent [1].

A. Resonant Length

Fig. 3 illustrates the variation of normalized resonant length, $2k_0 l_{res}$, as a function of the tilt angle θ for different wall thicknesses. The resonant slot length is $2l_{res}$, and k_0 is the free-space wavenumber. For standard height X-band waveguide, $2k_0 l_{res}$ is relatively insensitive to the tilt θ; an increase of only 0.5 percent can be observed for $t = 0.01$ in. and θ lying between 10° and 35°. For reduced height waveguides, $2k_0 l_{res}$ shows a significant increase with θ: 1.8 percent for $b = 0.2$ in. and 5 percent for $b = 0.1$ in. with $t = 0.01$ in. and with θ lying between 10° and 35°. As shown in Fig. 3, for thick wall slots, $2k_0 l_{res}$ shows less sensitivity to θ. Resonant length increases with t if $2l_{res} < \lambda/2$ whereas it decreases with t if $2l_{res} > \lambda/2$. Here λ is the free-space wavelength. A similar behavior for resonant length was reported for *radiating* slots in [10]. The normalized resonant length decreases at higher frequencies, as illustrated by Fig. 4. The reduction in $2k_0 l_{res}$ is about 3.5 percent over a 17 percent change in frequency for a standard height X-band waveguide, whereas it is 4 percent and 7 percent for half-height and quarter-height guides, respectively. The normalized resonant length shows less sensitivity to frequency for larger tilt angles and wall thicknesses. The quantity $2k_0 l_{res}$ shows a similar dependence on waveguide parameters, b and t, and on frequency for compound radiating slots [10] and for shunt–series coupling slots [12].

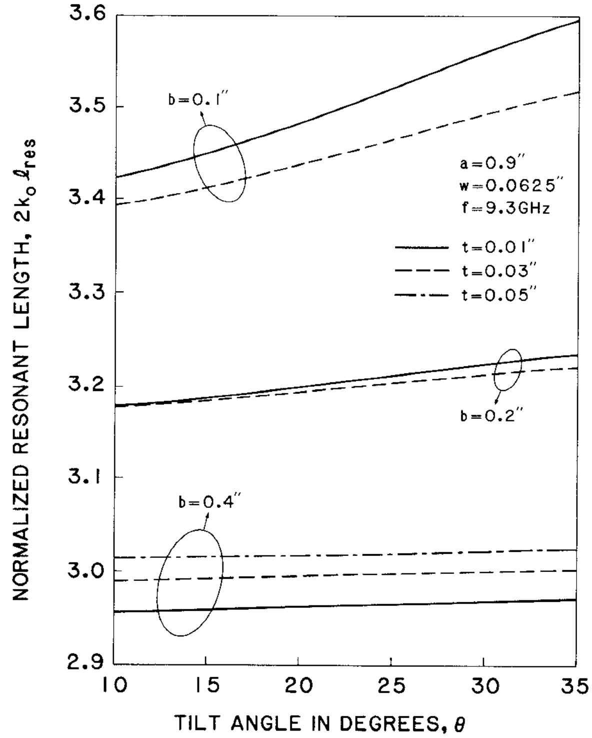

Fig. 3. Normalized resonant length as a function of tilt angle

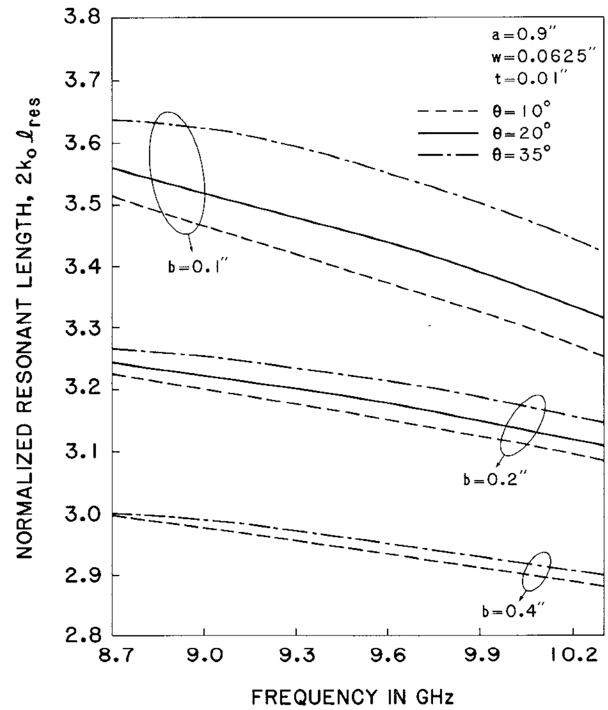

Fig. 4. Normalized resonant length as a function of frequency.

Fig. 5 shows the variation of resonant length, $2k_0 l_{res}$, as a function of tilt for the different slot widths $w = 1/32$ in., 1/16 in., and 3/32 in. For standard X-band waveguide, a wider slot is seen to be of shorter resonant length. However, a wider slot in reduced height waveguide exhibits a longer resonant length. The resonant length of a centered-inclined slot coupler is less than that of a shunt–series

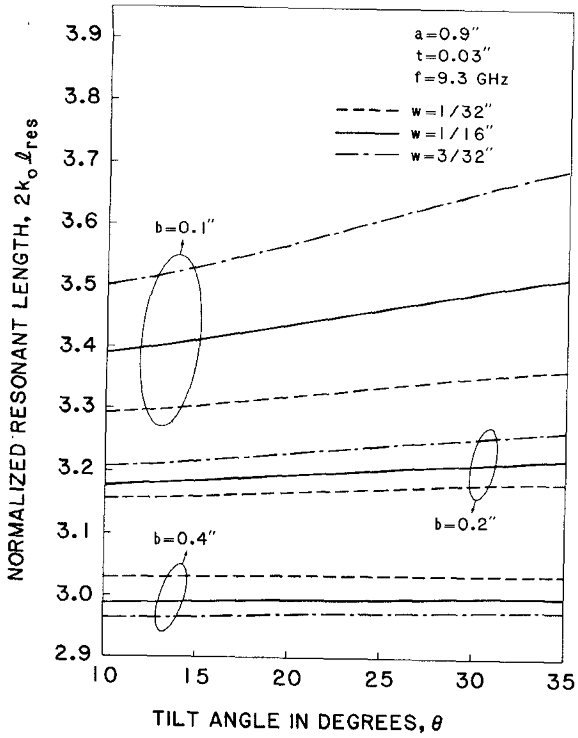

Fig. 5. Normalized resonant length for different slot widths.

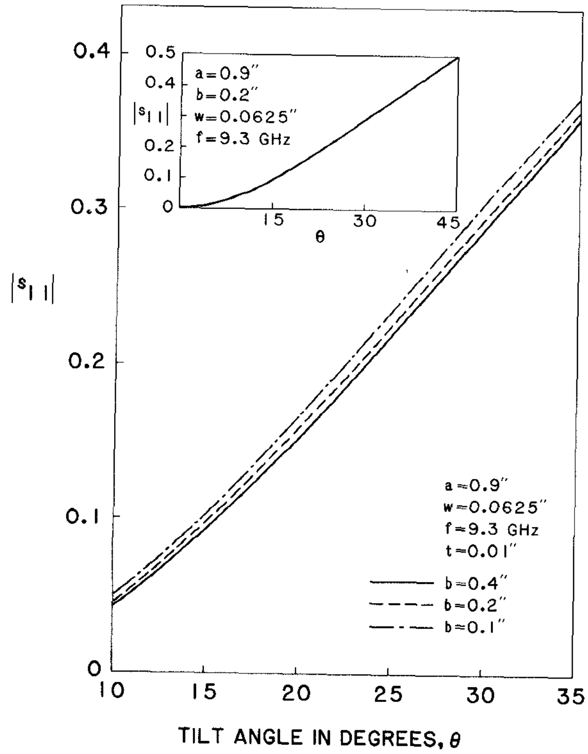

Fig. 6. $|S_{11}|$ at resonance as a function of tilt angle.

coupling slot [12], especially for greater coupling and a smaller b dimension.

B. The Backscattered Wave Amplitude, $|S_{11}|$

Fig. 6 shows the backscattered wave amplitude, $|S_{11}|$, *at resonance* as a function of the tilt angle for three different X-band waveguides. The dependence of $|S_{11}|$ at resonance on wall thickness and slot width is seen not to be significant. The waveguide b dimension affects $|S_{11}|$ by a small amount, as seen in Fig. 6. The inset in this figure shows the variation of $|S_{11}|$ between $\theta = 0°$ and $45°$. For all three cases $|S_{11}| = 0$ at $\theta = 0°$ and $|S_{11}| = 0.5$ at $\theta = 45°$. The dependence of $|S_{11}|$ *at resonance* on frequency is depicted in Fig. 7 for $\theta = 10°$, $20°$, and $35°$ and for $b = 0.1$ in. and 0.4 in. Over a frequency range of 17 percent $|S_{11}|$ at resonance drops by 22 percent at 10° and drops about 6 percent for 35° for standard height X-band waveguide. For quarter-height guide, these values are 30 percent and 8.5 percent, respectively. Clearly, as $\theta \to 45°$ $|S_{11}|$ at resonance becomes independent of frequency. In contrast, for a compound radiating slot, and for a shunt–series coupling slot, $|S_{11}|$ at resonance drops more rapidly with frequency [10], [12]. Centered-inclined coupling slots have better performance at higher frequencies when compared to shunt–series coupling slots, particularly for reduced height waveguides. In the latter case, to achieve a given coupling at a higher frequency, a greater offset and a longer slot (especially in reduced height guides) would be required.

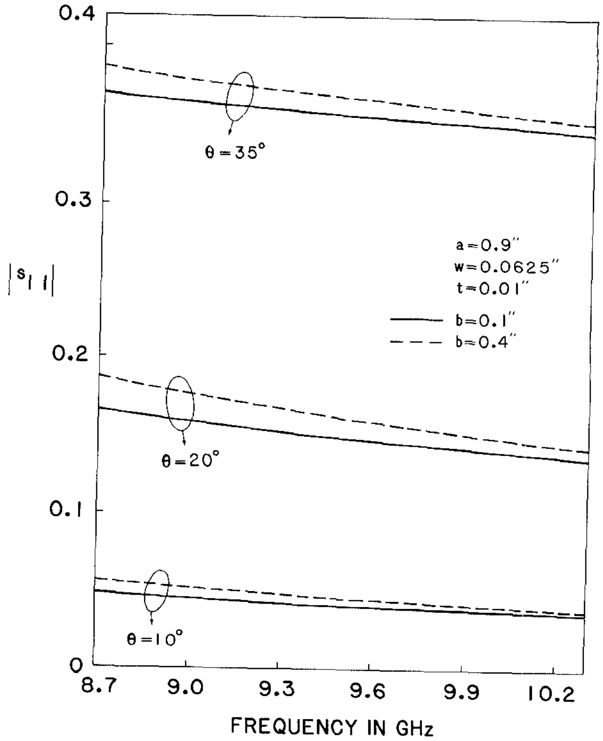

Fig. 7. $|S_{11}|$ at resonance as a function of frequency

C. Off-Resonant Characteristics

Fig. 8 shows the phase variation of S_{11} over a 2 percent frequency change from resonance. The phase variation of S_{11} over this frequency range is very nearly the same as for S_{31}, S_{41} and the forward scattered TE_{10} wave. Thick wall slots exhibit the greatest phase variation. Thus, thin wall slots have more desirable characteristics, thereby effecting improved performance in slot array. Fig. 9 illustrates a similar phase variation over a 1 percent frequency change from resonance for three different slot widths. Wider slots have smaller phase variation with frequency and hence are useful in a relatively broader frequency range. Reduced

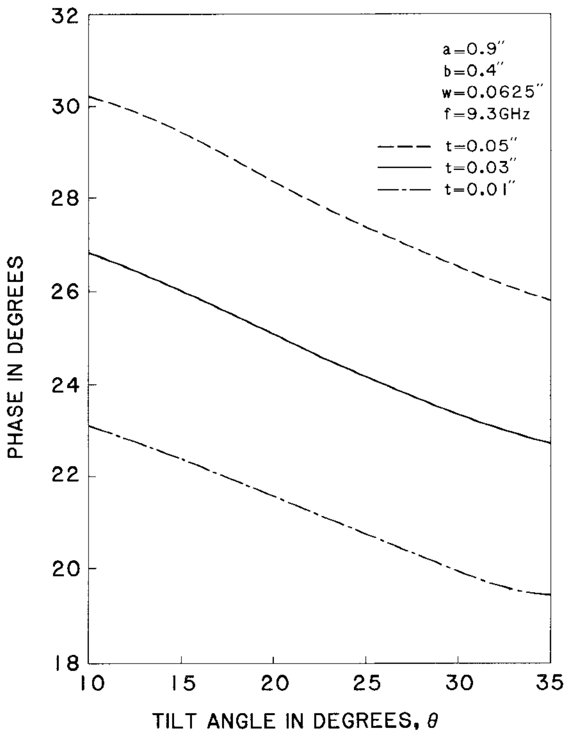

Fig 8. Phase variation of S_{11} over a 2 percent frequency change from resonance.

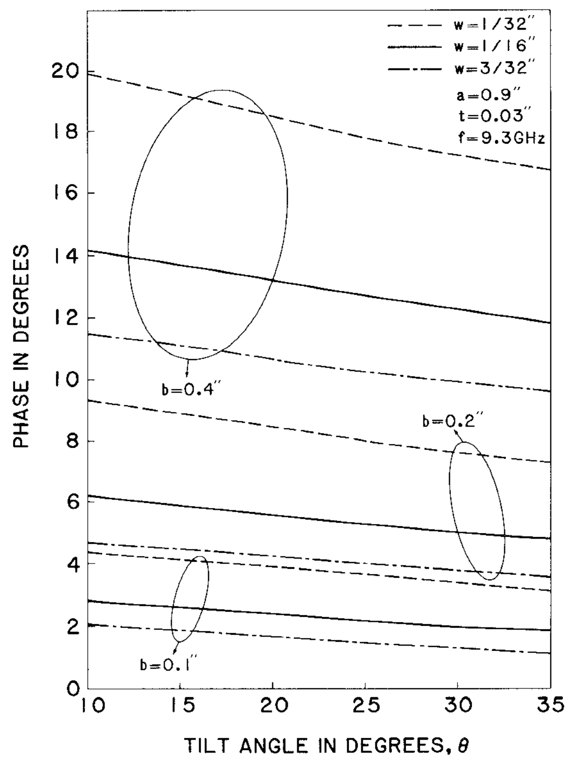

Fig. 9. Phase variation of S_{11} over a 1 percent frequency change from resonance for different slot widths.

height waveguide slot couplers possess a slowly varying phase characteristic off resonance. Quarter-height waveguide centered-inclined couplers have excellent phase characteristics. By contrast, shunt–series coupling slots exhibit a greater phase variation off resonance, in general, and their resonant length is long in reduced height waveguides [12].

D. *Effect of the Number of Galerkin Modes*

All the results discussed so far were obtained with three expansion modes in the global Galerkin technique. When the number of modes is increased to ten, only the values for resonant length show any noticeable change. Even this change is of the order of 0.5 percent for $t = 0.01$ in., and it is 0.33 percent and 0.25 percent for $t = 0.03$ in. and 0.05 in., respectively. The first expansion mode coefficient for the aperture electric field at resonance is about an order of magnitude greater than the third mode and it is about two orders of magnitude greater than the second. Higher order Galerkin modes are substantially less in magnitude. Away from resonance, higher order modes become more significant. The effect of higher order Galerkin modes is further reduced for increasing wall thickness and tilt. Thus results obtained with three Galerkin modes are usually quite satisfactory.

E. *Comparison of Theory and Experiment*

For experimental validation of this theory, a test fixture was made from two aluminum blocks. An open waveguide channel of 0.9 in.×0.2 in. inner dimensions was milled in each block such that when the blocks were bolted together, the channels were orthogonal to each other. A copper plate of 0.01 in. thickness was clamped between the blocks, forming a common wall separating the waveguides. A series of copper plates was made, each containing a square ended coupling slot at the center of the waveguide intersection. Three slot lengths were made for each tilt angle: 0.65 in., 0.66 in., and 0.67 in. The slots were etched in copper plates and their dimensions were checked with a microscope.

A transition from 0.9 in.×0.2 in. to 0.9 in.×0.4 in. waveguide and a coax-to-waveguide adapter were used to connect the coupling slot fixture to an HP 8510 automatic network analyzer (ANA). The ANA was calibrated using half-height waveguide fixed shorts and sliding loads. The reflection coefficient in the mainline, S_{11}, was measured over a range of frequencies. The resonant frequency at which the phase of S_{11} is zero was then determined for each slot. For each tilt angle a set of three data points (resonant frequencies for slot lengths of 0.65 in., 0.66 in., and 0.67 in.) were obtained. Using a linear interpolation of these data, the resonant length at the desired frequency of 9.17 GHz was determined.

A comparison between computed values for $|S_{11}|$ and $2l_{res}$ shown here and those obtained experimentally is made in Table I. The agreement between theory and experiment is seen to be excellent. The worst-case deviation in resonant length is 0.75 percent and that in $|S_{11}|$ is 6 percent. The greatest deviation for $|S_{11}|$ occurs for $\theta = 15°$. For this case, since the coupling is small, measurement errors, manufacturing tolerances, etc., can be significant. For larger tilt angles, the $|S_{11}|$ comparison is much better.

TABLE I
COMPARISON OF THEORY AND EXPERIMENT

	$2\ell_{res}$ in inch		$\|S_{11}\|$	
θ	Theory	Experiment	Theory	Experiment
15°	0.655	0.659	0.099	0.105
20°	0.657	0.662	0.161	0.165
25°	0.660	0.662	0.230	0.235
30°	0.662	0.665	0.299	0.307
35°	0.664	0.665	0.367	0.375

$a = 0.9$ in., $b = 0.2$ in., $w = 0.0625$ in., $t = 0.01$ in., $f = 9.17$ GHz.

It is not clear to what extent discrepancies are due to experimental errors or to minor imperfections in theory.

IV. Conclusions

Integral equations have been developed for a centered-inclined coupling slot, including the effect of finite wall thickness of the common broad wall, and they have been solved by the method of moments. Numerical results for resonant length, backscattered wave amplitude, and phase variation off resonance have been presented over a range of values of the waveguide b dimension, wall thickness, slot width, and frequency. These results have significant applications in the design of waveguide-fed slot arrays.

The resonant length is relatively insensitive to slot tilt θ for a standard height X-band waveguide, whereas its dependence on θ is significant for reduced height waveguides. The phase variation of scattered TE_{10} waves in both waveguides off resonance is less for wider slots and smaller b dimensions. Shunt–series coupling slots exhibit greater phase variation off resonance when compared to a centered-inclined coupling slot. Also, the former has a longer resonant length for a smaller b dimension and for a wider slot. Thus the centered-inclined slot coupler possesses superior characteristics.

The higher order mode coupling between a centered-inclined slot coupler and a pair of straddling radiating slots in the branch waveguide is very significant. This coupling effect should be properly taken into account in the design of waveguide-fed slot arrays. This problem is presently under investigation at UCLA.

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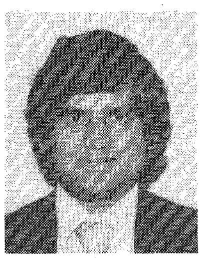

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